THURSDAY, MAY 10, 1877

## MATHEMATICS IN AMERICA

*Elements of the Differential and Integral Calculus, by a new Method, founded on the True System of Sir Isaac Newton, without the Use of Infinitesimals or Limits.* By C. P. Buckingham. (Chicago: S. C. Griggs and Co., 1875. 343 pp.)

*Elements of the Infinitesimal Calculus, with Numerous Examples and Applications to Analysis and Geometry.* By James G. Clark, A.M. (Ray Series. New York: Wilson, Hinkle, and Co., 1875. 441 pp.)

*On a New Method of Obtaining the Differentials of Functions with Especial Reference to the Newtonian Conception of Rates or Velocities.* By J. Minot Rice and W. Woolsey Johnson. (New York: D. van Nosbrand, 1875. 32 pp.)

AN American writer who had exceptional opportunities of contrasting the methods of mathematical teaching adopted in his own country with those which obtained at Cambridge twenty-five years ago, strongly condemns the Transatlantic system, and leads his readers to infer that the attainments of the ordinary graduate in this particular branch of study were only on a par with those of a fairly trained schoolboy here. It may be supposed, then, that not many of the students ventured upon the difficulties of the calculus. Indeed, he writes that "at Yale where the course used to be thought a very difficult and thorough one, the Differential was among the *optional* studies at the end of the third year." (Bristed: "Five Years at an English University," vol. ii., pp. 94, &c., 1852.)

We are not in a position to say that all this has been changed in the interim, but among many evidences of the increased interest taken in mathematical studies we may surely refer to the three works now before us. All three give evidence of careful study and honestly grapple with the difficulties which beset the learner at the very threshold of his inquiries. De Morgan long ago wrote that "it is matter of common observation that any one who commences the study, even with the best elementary works, finds himself in the dark as to the real meaning of the processes which he learns, until, at a certain stage of his progress, depending upon his capacity, some accidental combination of his own ideas throws light upon the subject." The authors of the third work under review refer to D'Alembert's precept, "Allez en avant, et la foi vous viendra."

Mr. Buckingham takes as his fundamental idea of the conditions under which quantity may exist to be that we must not consider it only as *capable* of being increased or diminished, but also as being actually in a *state of change*. "It must (so to speak) be *vitalised*, so that it shall be endowed with *tendencies* to change its value; and the rate and direction of these tendencies will be found to constitute the groundwork of the whole system. The differential calculus is the SCIENCE OF RATES, and its peculiar subject is QUANTITY IN A STATE OF CHANGE."

Conceding to Leibnitz the honour of being the first to construct a system of rules for the *analytical machinery* of the science, he will not allow that he ever got beyond the ancient conception of the conditions of quantity. "The only original birthplace of the fundamental idea of quantity which forms the true germ of the calculus was in the mind of the immortal Newton."

An introduction of thirty-six pages discusses the method of Descartes, the infinitesimal method (the results of which are true, while the method is false—"true results *not* because its principles are true, nor because its errors are small, but because they are, whether great or small, exactly equal, and *exactly cancel and destroy each other*. . . . the system is but a mere artifice."), the method of limits (here our author discusses Lemma I., Book I. of the "Principia," considers Newton's defence of the Lemma, and the opinions of Comte, Lagrange, and Berkeley, and points out what he believes to be the fundamental errors of this method and of the infinitesimal method). What is called the true method of Newton is then treated of. Referring to Newton's letter to J. Collins (December 10, 1672), he says that the theory on which Newton formed *his* method of fluxions is contained in the second Lemma. The lemma is given in full and discussed. "It is to be remarked that the doctrine of *limits* is nowhere hinted at, but the results are direct, positive, and substantial." We cannot tarry longer over this matter, but in connection with this point refer to De Morgan's "On the Early History of Infinitesimals in England" (*Phil. Mag.* November 1852). Prof. Clifford, too, if our recollection of an oral communication be correct, puts this lemma prominently forward in his (? unpublished) "Foundations of the Differential Calculus and of Dynamics." In the work itself we have the calculi (differential and integral) applied to the subjects which usually find a place in similar treatises. There is an appendix of thirteen pages on geometrical fluxions. Many examples are worked out, but the merit of the work does not lie at all, we think, in this direction, but altogether in the numerous discussions which are to be found in almost every chapter.

Mr. Clark's work has been exceedingly well printed, the type is very clear, and the paper good. This treatise, too, is written with a view to remove "all grounds for that feeling of uncertainty which often possesses the student at the very outset, and from which he rarely finds it possible to extricate himself." Much space is given to an exposition of the Doctrine of Limits—the work being founded mainly on that by Duhamel. A large number of examples have been taken from English treatises (Hall, Walton, and Todhunter). Rather more ground is covered in this treatise than in the former; in neither, however, have we any discussion of maxima and minima of functions of more than two independent variables, nor of methods of changing the variables in multiple integrals. Here a few pages are devoted to definite integrals and to differentiation and integration under the sign $\int$. Seven chapters are devoted to the elementary parts of the theory of differential equations. The work, though it does not reach the level of the like works by Messrs. Todhunter and Williamson, is yet a compact and fair elementary treatise.

The third work on our list is a revised edition of a paper read before the American Academy of Arts and Sciences, January 14, 1873. It is the authors' intention

to publish a text-book in accordance with the plan adopted in this pamphlet. The objects are, "first to present a new method of deriving the differentials of functions by means of their algebraic characteristics with the aid of a few elementary properties easily established, and secondly to show that the method of rates or fluxions may be advantageously used for the purposes of instruction, and the use of infinitesimals, limits, and series entirely avoided until the student is well grounded in the elements of the calculus."

The first seven articles under the head "the Newtonian Method of Fluxions," treat of the methods in general use at the present time, and contain extracts from Todhunter, Lacroix, Carnot, and Cournot, especially directing attention to the positive advantages of the Newtonian method, as set forth by the last-named writer. The next six articles are occupied with the "Proposed Method of treating the Differential Calculus."

The remaining half of the pamphlet is given to algebraic and transcendental functions. It would be very interesting to lay before our readers an account of the ingenious methods adopted by our authors, but it would take up too much space. Some idea of the original paper (and there are no great differences, we fancy, between the two publications) can be got from an account of it furnished by Mr. J. W. L. Glaisher, F.R.S., in vol. iv. (pp. 58-64) of the *Messenger of Mathematics* (1875).

Altogether, on a review of the three books before us, we anticipate that mathematical studies are destined to occupy a more prominent position in the American colleges and schools than they have in the past.[1]

R. TUCKER

[1] We are confirmed in our views on this subject by a perusal of Dr. Sylvester's characteristic address at the Johns Hopkins University on Commemoration Day, February 22, 1877.

## LETTERS TO THE EDITOR

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts. No notice is taken of anonymous communications.*

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### Visibility of the Ultra-Violet Rays of the Spectrum

It is well known how surprisingly rich in rays of high refrangibility the spectrum of the electric arc formed between carbon points is, above that of all other artificial flames; and also how far beyond the ordinarily discernible rays of the solar spectrum, formed by a glass prism, light may be traced by eyes carefully shielded, and raised to the highest susceptibility to perceive it. The name of "lavender-grey" rays has been given to them from a colour of that tint which they are considered to possess, but the term "ultra-violet," which is more commonly used, betrays perhaps a lingering doubt as to the sensible existence of another order of coloured rays in the spectrum distinct from the violet and superior to it in refrangibility, which has yet been detected by very close and careful observation. All doubts of this kind, which from want of sufficient acquaintance with that part of the spectrum I have myself been hitherto rather too prone to entertain, have lately been quite dispelled by frequent observations of the spectrum of the electric arc between carbon points thrown by a quartz prism on a white paper screen. The violet end of the spectrum terminates rather abruptly, or at least beams with bright colour that fades off very quickly; and in the dark space beyond it three more refrangible bright bands are visible with more or less distinctness. The middle one of the three is the brightest, and from its perfect freedom from colour, in which it contrasts most remarkably with the strongly-tinted light-belt near it, and its distant separation from the violet termination of the continuous spectrum, I at first hastily ascribed it to a "ghost," or faint image of the slit, indirectly refracted and reflected through the prism, and thrown with the spectrum on the screen. That it is not so, however, is shown by the action of these three lines on fluorescent substances, of all of which that I have tried they excite the fluorescence most strongly, especially that of fluorescin, eosin, rose of Magdala, and other solutions, all of which alike show these rays to be clearly defined and well-insulated spectral bands. In particular, the solutions of æsculin, paviin and amido-terephthalic acid are only excited by these "ultra-violet" lines, and not by any rays in the spectrum of lower refrangibilities, clearly showing that the vigorous fluorescence that they produce is not the effect of any ordinary light-beam of common refrangibility, irregularly transmitted by the prism, but that they are well-marked rays, probably of carbon, in the spectrum of the voltaic arc. The light of the middle band is bright enough to be easily reflected and examined separately from the rest of the spectrum on a white screen, where it is so nearly grey or colourless that it scarcely admits of being ranged in any colour scale, although the name "lavender-grey" perhaps expresses better than any other term the faintest possible tone of colour which, if it exhibits any at all, this almost purely neutral, or steel-grey band of rays may possibly be suspected to possess. It is a little more strongly absorbed by ordinary plate-glass than the neighbouring violet bands; but it remains visible in the spectrum of the arc formed by an ordinary flint glass prism, though much spread out and enfeebled by the dispersion, which greatly exceeds that of a quartz prism of the same refracting angle. It is perhaps for this reason that it is not perceptible in the spectrum of the arc as usually projected on a screen with a fluid-prism of bisulphide of carbon, but if the latter is replaced by benzine, which disperses the light less than flint glass, it forms a pretty conspicuous grey band in the spectrum. The other two lines or bands are so much fainter than the principal one, that in general they can only be found with the help of a fluorescent substance, and where so faintly visible it is not possible to speak positively as to their colour. The less refrangible is very near the violet termination of the spectrum, and when well seen it shares its violet tinge; the more refrangible one is nearly as far beyond the principal grey band as this band is beyond the margin of the violet, and as far as its weak light allows one to distinguish, it is of the same colour as the brightest band. In order to determine their positions, some measurements were made of metallic lines, and of the spectra of sodium, lithium, thallium, and strontium in the arc, with the result that the violet part of the continuous spectrum extends to the closely-neighbouring positions of the hydrogen line H$\delta$ ($h$), the potassium flame-spectrum line K$\beta$, and the last violet line in the arc spectrum of a salt of strontium. The first faint outlying ray occupies nearly the position of $H_1$ in the solar spectrum, and it is therefore in the true violet region of the spectrum, as its colour faintly indicates. The prominent grey line begins with its brightest edge about as much further beyond this, from the end of the violet field; and becoming weaker from there, it is about twice as broad as the distance between the two Fraunhofer lines H, its mean position in the spectrum being nearly as far from H as H is from $h$, reckoning the distances as they would be seen with the quartz prism and with solar light. The third faint line occurs about as far again from the violet as this band; and it lies at least as far beyond H as the distance between G and H in the solar spectrum. Yet it is visible there by glimpses, like the first faint member of the group, which it does not yet by any means surpass in the strength with which it produces fluorescence.

If any fresh proof was needed of the characteristic grey appearance of visible rays in this portion of the spectrum it was soon presented in one of the metallic spectra used to determine their positions. The spectrum of mercury exhibited a bright line (beautifully distinct when a fluid prism of benzine was used with a refracting angle of between 50° and 60°), much brighter than the principal grey carbon band, considerably more refrangible, and of the same tintless, and perfectly neutral grey appearance. Though not so distant from the violet as the most refrangible faint carbon line, it is yet according to the best measurements and identifications that were made, about as far beyond $H_1$ in a prismatic spectrum as $H_1$ is from G; and radiation of this high degree of refrangibility is evidently strongly luminous, when sufficiently intense, with homogeneous grey light characteristic of this region, and contrasting conspicuously in its appearance with the zone of violet colour, which often borders closely upon it in electric spectra.